MCM

MARIANISTAS + COMPAÑÍA DE MARÍA

I. - TRIGONOMETRÍA

FÓRMULAS BÁSICAS

$$\operatorname{sen}\alpha = \frac{c}{a}$$

$$\operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha} = \frac{a}{c}$$

$$\cos\alpha = \frac{b}{a}$$

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{a}{b}$$

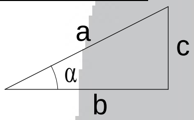
$$\tan\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{c}{b}$$

$$\operatorname{cotan}\alpha = \frac{1}{\tan\alpha} = \frac{b}{c}$$

$$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$$

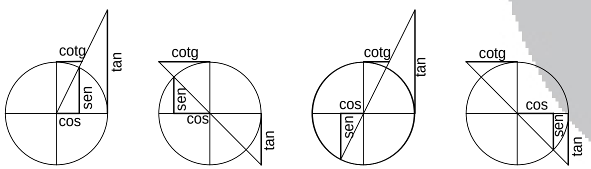
$$\tan\alpha \times \operatorname{cotan}\alpha = 1$$

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} = \sec^2\alpha$$



$$1 + \operatorname{cotan}^2\alpha = \frac{1}{\operatorname{sen}^2\alpha} = \operatorname{cosec}^2\alpha$$

LINEAS TRIGONOMÉTRICAS



Primer Cuadrante — Segundo Cuadrante — Tercer Cuadrante — Cuarto Cuadrante

Ángulos complementarios:

DETERMINACION DE UNA RAZON EN FUNCION DE OTRA

En función del seno:

$$\operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha}$$

$$\cos\alpha = \sqrt{1 - \operatorname{sen}^2\alpha}$$

$$\sec\alpha = \frac{1}{\sqrt{1 - \operatorname{sen}^2\alpha}}$$

$$\tan\alpha = \frac{\operatorname{sen}\alpha}{\sqrt{1 - \operatorname{sen}^2\alpha}}$$

$$\operatorname{ctg}\alpha = \frac{\sqrt{1 - \operatorname{sen}^2\alpha}}{\operatorname{sen}\alpha}$$

En función del coseno:

$$\operatorname{sen}\alpha = \sqrt{1 - \cos^2\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha}$$

$$\operatorname{cosec}\alpha = \frac{1}{\sqrt{1 - \cos^2\alpha}}$$

$$\tan\alpha = \frac{\sqrt{1 - \cos^2\alpha}}{\cos\alpha}$$

$$\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sqrt{1 - \cos^2\alpha}}$$

En función de la tangente:

$$\cos\alpha = \frac{1}{\sqrt{1 + \tan^2\alpha}}$$

$$\operatorname{ctg}\alpha = \frac{1}{\tan\alpha}$$

En función del coseno del ángulo doble:

(Usadas para integrar)

$$\operatorname{sen}\alpha = \sqrt{\frac{1 - \cos 2\alpha}{2}}$$

$$\operatorname{sen}\frac{\alpha}{2} = \sqrt{\frac{1 - \cos\alpha}{2}}$$

$$\cos\alpha = \sqrt{\frac{1 + \cos 2\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1 + \cos\alpha}{2}}$$

$$\tan\alpha = \sqrt{\frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}}$$

$$\tan\frac{\alpha}{2} = \sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}}$$

RAZONES DEL ÁNGULO SUMA/DIFERENCIA

$$\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha \cos\beta \pm \cos\alpha \operatorname{sen}\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$$

$$\frac{\operatorname{sen}\alpha + \operatorname{sen}\beta}{\operatorname{sen}\alpha - \operatorname{sen}\beta} = \frac{\tan\frac{1}{2}(\alpha + \beta)}{\tan\frac{1}{2}(\alpha - \beta)}$$

$$\operatorname{ctg}(\alpha \pm \beta) = \frac{\operatorname{ctg}\alpha \operatorname{ctg}\beta \mp 1}{\operatorname{ctg}\alpha \pm \operatorname{ctg}\beta}$$

$$\frac{\cos\alpha + \cos\beta}{\cos\alpha - \cos\beta} = -\frac{\alpha + \beta}{2} \operatorname{cotan}\frac{\alpha - \beta}{2}$$

TRANSFORMACION DE SUMAS A PRODUCTOS Y VICEVERSA

(Estas expresiones se utilizan en la resolución de triángulos con el empleo de logaritmos)

SUMAS a PRODUCTOS

$$\operatorname{sen}\alpha + \operatorname{sen}\beta = 2\operatorname{sen}\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$$

$$\operatorname{sen}\alpha - \operatorname{sen}\beta = 2\cos\frac{\alpha + \beta}{2}\operatorname{sen}\frac{\alpha - \beta}{2}$$

REDUCCION AL 1er CUADRANTE

Su suma vale π/2 radianes (90º)

$\operatorname{sen}(\pi/2-\alpha)=\cos\alpha$
$\cos(\pi/2-\alpha)=\operatorname{sen}\alpha$
$\tan(\pi/2-\alpha)=\operatorname{ctg}\alpha$

Ángulos suplementarios:
Su suma vale π radianes (180º)

$\operatorname{sen}(\pi-\alpha)=\operatorname{sen}\alpha$
$\cos(\pi-\alpha)=-\cos\alpha$
$\tan(\pi-\alpha)=-\tan\alpha$

Ángulos que difieren en π/2 radianes:

$\operatorname{sen}(\pi/2+\alpha)=\cos\alpha$
$\cos(\pi/2+\alpha)=-\operatorname{sen}\alpha$
$\tan(\pi/2+\alpha)=-\operatorname{ctg}\alpha$

Ángulos que se diferencian π radianes:

$\operatorname{sen}(\pi+\alpha)=-\operatorname{sen}\alpha$
$\cos(\pi+\alpha)=-\cos\alpha$
$\tan(\pi+\alpha)=\tan\alpha$

Ángulos opuestos:

$\operatorname{sen}(-\alpha)=-\operatorname{sen}(\alpha)$
$\cos(-\alpha)=\cos\alpha$
$\tan(-\alpha)=-\tan\alpha$

$$\sec\alpha=\sqrt{1+\tan^2\alpha}$$

$$\operatorname{cosec}\alpha=\frac{\sqrt{1+\tan^2\alpha}}{\tan\alpha}\qquad \operatorname{sen}\alpha=\frac{\tan\alpha}{\sqrt{1+\tan^2\alpha}}$$

En función de la tangente del ángulo mitad
(Usadas para integrar)

$$\operatorname{sen}\alpha=\frac{2\tan(\alpha/2)}{1+\tan^2(\alpha/2)}\qquad \operatorname{sen}2\alpha=\frac{2\tan\alpha}{1+\tan^2\alpha}$$

$$\cos\alpha=\frac{1-\tan^2(\alpha/2)}{1+\tan^2(\alpha/2)}\qquad \cos2\alpha=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

$$\tan\alpha=\frac{2\tan(\alpha/2)}{1-\tan^2(\alpha/2)}\qquad \tan2\alpha=\frac{2\tan^2\alpha}{1-\tan^2\alpha}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\qquad \cos\alpha-\cos\beta=-2\operatorname{sen}\frac{\alpha+\beta}{2}\operatorname{sen}\frac{\alpha-\beta}{2}$$

$$\tan\alpha+\tan\beta=\frac{\operatorname{sen}(\alpha+\beta)}{\cos\alpha\cos\beta}$$

$$\tan\alpha-\tan\beta=\frac{\operatorname{sen}(\alpha+\beta)}{\cos\alpha\cos\beta}$$

$$\operatorname{ctg}\alpha+\operatorname{ctg}\beta=\frac{\operatorname{sen}(\alpha+\beta)}{\operatorname{sen}\alpha\operatorname{sen}\beta}$$

$$\operatorname{ctg}\alpha-\operatorname{ctg}\beta=\frac{\operatorname{sen}(\beta-\alpha)}{\operatorname{sen}\alpha\operatorname{sen}\beta}$$

PRODUCTOS a SUMAS

$$\operatorname{sen}\alpha\operatorname{sen}\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\operatorname{sen}\alpha\cos\beta=\frac{1}{2}\left[\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)\right]$$

$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

FUNCIONES DE LOS MÚLTIPLOS DE UN ÁNGULO

Ángulo doble

$$\operatorname{sen}2\alpha=2\operatorname{sen}\alpha\cos\alpha$$
$$\cos2\alpha=\cos^2\alpha-\operatorname{sen}^2\alpha$$
$$\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

Ángulo triple

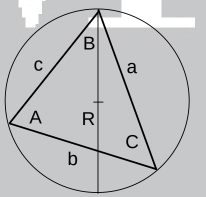
$$\operatorname{sen}3\alpha=3\operatorname{sen}\alpha-4\operatorname{sen}^3\alpha$$
$$\cos3\alpha=4\cos^3\alpha-3\cos\alpha$$
$$\tan3\alpha=\frac{3\tan\alpha}{1-3\tan^2\alpha}$$

FUNCIONES TRIGONOMÉTRICAS INVERSAS

$$\operatorname{arc\,sen}x=\arccos\sqrt{1-x^2}=\frac{\pi}{2}-\arccos x$$

$$\operatorname{arc\,cos}x=\operatorname{arcsen}\sqrt{1-x^2}=\frac{\pi}{2}-\operatorname{arcsen}x$$

TEOREMAS IMPORTANTES:



Teorema de los senos:

$$\frac{a}{\operatorname{sen}A}=\frac{b}{\operatorname{sen}B}=\frac{c}{\operatorname{sen}C}=2R$$

Teorema de los cosenos:

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}$$

$$\cos C=\frac{a^2+b^2-c^2}{2ab}$$

Teorema de las tangentes

FUNCIONES DEL ÁNGULO SUMA/DIFERENCIA

$$\operatorname{Sh}(\alpha+\beta)=\operatorname{Sh}\alpha\operatorname{Ch}\beta+\operatorname{Sh}\beta\operatorname{Ch}\alpha\qquad \operatorname{Sh}(\alpha-\beta)=\operatorname{Sh}\alpha\operatorname{Ch}\beta-\operatorname{Sh}\beta\operatorname{Ch}\alpha$$

$$\operatorname{Ch}(\alpha+\beta)=\operatorname{Ch}\alpha\operatorname{Ch}\beta+\operatorname{Sh}\beta\operatorname{Sh}\alpha\qquad \operatorname{Ch}(\alpha-\beta)=\operatorname{Ch}\alpha\operatorname{Ch}\beta-\operatorname{Sh}\alpha\operatorname{Sh}\beta$$

$$\operatorname{Th}(\alpha+\beta)=\frac{\operatorname{Th}\alpha+\operatorname{Th}\beta}{1+\operatorname{Th}\alpha\operatorname{Th}\beta}\qquad \operatorname{Th}(\alpha-\beta)=\frac{\operatorname{Th}\alpha-\operatorname{Th}\beta}{1-\operatorname{Th}\alpha\operatorname{Th}\beta}$$

FUNCIONES DEL ÁNGULO DOBLE/MITAD

$$\operatorname{Sh}2\alpha=2\operatorname{Sh}\alpha\operatorname{Ch}\alpha\qquad \operatorname{Ch}2\alpha=\operatorname{Sh}^2\alpha+\operatorname{Ch}^2\alpha\qquad \operatorname{Th}2\alpha=\frac{2\operatorname{Sh}\alpha\operatorname{Ch}\alpha}{\operatorname{Sh}^2\alpha+\operatorname{Ch}^2\alpha}$$

$$\operatorname{Sh}\frac{\alpha}{2}=\sqrt{\frac{1}{2}(\operatorname{Ch}\alpha-1)}\qquad \operatorname{Ch}\frac{\alpha}{2}=\sqrt{\frac{1}{2}(\operatorname{Ch}\alpha+1)}\qquad \operatorname{Th}\frac{\alpha}{2}=\sqrt{\frac{\operatorname{Ch}\alpha-1}{\operatorname{Ch}\alpha+1}}$$

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MARIANISTAS + COMPAÑÍA DE MARÍA

$$\arctan x = \operatorname{arcsen} \frac{x}{\sqrt{1+x^2}} = \frac{\pi}{2} - \operatorname{arctg} x$$

$$\operatorname{arcsen} x + \operatorname{arcsen} y = \operatorname{arcsen}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$$

$$\operatorname{arcsen} x - \operatorname{arcsen} y = \operatorname{arcsen}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$$

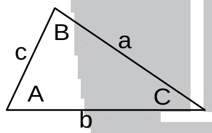
$$\arccos x + \arccos y = \arccos\left[xy - \sqrt{(1-x^2)(1-y^2)}\right]$$

$$\arccos x - \arccos y = \arccos\left[xy + \sqrt{(1-x^2)(1-y^2)}\right]$$

$$\arctan x + \arctan y = \frac{x+y}{1-xy} \qquad \arctan x - \arctan y = \frac{x-y}{1+xy}$$

FÓRMULAS DE BRIGGS

Para las tangentes de los ángulos mitad, se dividen las expresiones análogas miembro a miembro. Para el ángulo entero se utilizan las fórmulas que dan
las razones de un ángulo en función del coseno del ángulo doble. Estas fórmulas ya se han tratado anteriormente.



$$\operatorname{sen}\frac{A}{2} = \sqrt{\frac{(p-b)(p-c)}{bc}}$$

$$\operatorname{sen}\frac{B}{2} = \sqrt{\frac{(p-a)(p-c)}{ac}} \qquad \cos\frac{B}{2} = \sqrt{\frac{p(p-b)}{ac}}$$

$$\operatorname{sen}\frac{C}{2} = \sqrt{\frac{(p-b)(p-a)}{ab}} \qquad \cos\frac{C}{2} = \sqrt{\frac{p(p-c)}{ab}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{p(p-c)}{bc}} \qquad p = \frac{a+b+c}{2}$$

$$\frac{a+b}{a-b} = \frac{\operatorname{sen} A + \operatorname{sen} B}{\operatorname{sen} A - \operatorname{sen} B} = \frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$$

AREA DEL TRIÁNGULO

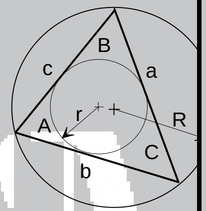
$$S = \frac{1}{2}ab\operatorname{sen} C = \frac{1}{2}cb\operatorname{sen} A = \frac{1}{2}ac\operatorname{sen} B$$

(Fórmula de Herón)

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

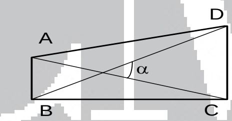
$$S = pr = \frac{abc}{4R} = \frac{p}{2R}$$

$$p = \frac{a+b+c}{2}$$



AREA DE UN CUADRILÁTERO

$$S = \frac{\overline{AC}\ \overline{BD}}{2}\operatorname{sen}\alpha$$



II.- FUNCIONES HIPERBÓLICAS

FÓRMULAS BÁSICAS

$$\operatorname{Sh}\alpha = \frac{e^{\alpha} - e^{-\alpha}}{2}$$

$$\operatorname{Ch}\alpha = \frac{e^{\alpha} + e^{-\alpha}}{2}$$

$$\operatorname{Th}\alpha = \frac{\operatorname{Sh}\alpha}{\operatorname{Ch}\alpha} = \frac{e^{\alpha} - e^{-\alpha}}{e^{\alpha} + e^{-\alpha}}$$

$$\operatorname{Sh}\alpha + \operatorname{Ch}\alpha = e^{\alpha}$$

$$\operatorname{Ch}\alpha - \operatorname{Sh}\alpha = e^{-\alpha}$$

TRANSFORMACION DE PRODUCTOS A SUMAS

$$\operatorname{Sh}\alpha\operatorname{Sh}\beta = \frac{1}{2}\left[\operatorname{Ch}(\alpha+\beta) - \operatorname{Ch}(\alpha-\beta)\right] \qquad \operatorname{Ch}\alpha\operatorname{Ch}\beta = \frac{1}{2}\left[\operatorname{Ch}(\alpha+\beta) + \operatorname{Ch}(\alpha-\beta)\right]$$

$$\operatorname{Sh}\alpha\operatorname{Ch}\beta = \frac{1}{2}\left[\operatorname{Sh}(\alpha+\beta) + \operatorname{Sh}(\alpha-\beta)\right] \qquad (\operatorname{Ch}\alpha \pm \operatorname{Sh}\alpha)^n = \operatorname{Ch} n\alpha \pm \operatorname{Sh} n\alpha$$

FUNCIONES HIPERBÓLICAS INVERSAS

$$\operatorname{ArgSh} x = \ln\left(x + \sqrt{x^2+1}\right) \qquad \operatorname{ArgCh} x = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\operatorname{ArgTh} x = \frac{1}{2}\ln\frac{1+x}{1-x} \qquad \operatorname{ArgSh} x = \operatorname{ArgCh}\sqrt{x^2+1} = \operatorname{ArgTh}\frac{x}{\sqrt{x^2+1}}$$

$$\operatorname{ArgCth} x = \frac{1}{2}\ln\frac{x+1}{x-1} \qquad \operatorname{ArgCh} x = \operatorname{ArgSh}\sqrt{x^2-1} = \operatorname{ArgTh}\frac{\sqrt{x^2-1}}{x}$$

$$\operatorname{ArgTh} x = \operatorname{ArgSh}\frac{x}{\sqrt{1-x^2}} = \operatorname{ArgCh}\frac{x}{\sqrt{1-x^2}} = \operatorname{ArgCth}\frac{1}{x}$$

RELACIONES ENTRE FUNCIONES CIRCULARES E HIPERBÓLICAS

$$\operatorname{sen} x = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right) \qquad \cos x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right)$$

$$\operatorname{Sh} ix = i\operatorname{sen} x \qquad \operatorname{sen} ix = i\operatorname{Sh} x \qquad e^{ix} = \cos x + i\operatorname{sen} x$$

$$\operatorname{Ch} ix = \cos x \qquad \cos ix = \operatorname{Ch} x \qquad e^{-ix} = \cos x - i\operatorname{sen} x$$

$$\operatorname{Th} ix = i\tan x \qquad \tan ix = i\operatorname{Th} x \qquad \operatorname{arcsen} ix = i\operatorname{Arg Sh} x$$

$$\operatorname{sen}(x+iy) = \operatorname{sen} x\operatorname{Ch} y + i\cos x\operatorname{Sh} y \qquad \arccos ix = -i\operatorname{Arg Ch} x$$

$$\cos(x+iy) = \cos x\operatorname{Ch} y - i\operatorname{sen} x\operatorname{Sh} y \qquad \arctan ix = i\operatorname{Arg Th} x = \frac{1}{2}i\ln\frac{1+x}{1-x}$$

$Ch^2\alpha - Sh^2\alpha = 1$